NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BHAM	**LEVEL:** 8
COURSE CODE: ANA801S	**COURSE NAME:** APPLIED NUMERICAL ANALYSIS
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS
1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencils.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [20 MARKS]

1.1 Consider the Simpson's rule:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] \tag{1.1}$$

State the extended rule for (1.1) where

$$h = \frac{(b-a)}{n};\ x_j = a + jh \quad \text{for each} \quad j = 0, 1, \dots, n$$ **[1.5]**

1.2 Hence apply the extended Simpson's rule to approximate the following integral (correct to 4 decimal places):

$$\int_a^b f(x)dx = \int_0^4 2.5e^x \sin(x)dx \tag{1.2}$$

using the sub-intervals [0, 1],[1, 2],[2, 3], and [3, 4] (i.e. when $h = \frac{1}{2}$).

Obtain the exact integral for (1.2) and hence determine the errors when using (1.1) and the extended rule version, stating the better approximation. **[23.5]**

QUESTION 2 [25 MARKS]

Consider the 2-point Gaussian quadrature rule:

$$\int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) \tag{2.1}$$

2.1 Show that the weights and the points in (2.1) are given by:

$$c_1 = \frac{b-a}{2}, \quad c_2 = \frac{b-a}{2}$$

$$x_1 = \left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right) + \frac{b+a}{2}, \quad x_2 = \left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$$ **[19]**

2.2 Hence obtain the Gaussian 2-point approximation for the integral (1.2) in Question 1 and compare your solutions with the Simpson's rule and the extended Simpson's rule obtained in Question 1. **[6]**

QUESTION 3 [25 MARKS]

3.1 Discuss and derive the recursive scheme for the Forward Euler's Method, using any appropriate diagram for substantiating your discussion. **[13]**

3.2 Consider the following IVP:

$$\frac{dy(t)}{dt} + 2y(t) = 3e^{-4t}, \quad y(0) = 1$$

Using a step size of h = 0.1 and $t_o = 0$, employ the method discussed in (3.1) to approximate up to the 5th step, giving your solution in a table showing both the exact and the approximate solution at each step. **[12]**

QUESTION 4 [30 MARKS]

4.1 Discuss with the aid of a diagram the 4th order Runge-Kutta (RK4) method **[16]**

4.2 Consider the following IVP:

$$\left.\begin{aligned} \frac{dy}{dx} &= xy \\ y(1) &= 5 \end{aligned}\right\} \quad (4.1)$$

Employing the RK4 method and using step size h = 0.1, solve (4.1) correct to three decimal places in the interval [1, 1.5] **[14]**

END OF QUESTION PAPER TOTAL MARKS = 100